MATH 1300, Mathematical Explorations

Combinations, Probability, and Small Numbers

Activity

- Birthday Paradox
  - Draw partial table and have them guess probability two people share a birthday.

# people	Probability of a match
1	0%
10	11.7%
20	41.1%
# students in the class	
100	99.99997%
200	99.99999999999999999999999998% (27 9's after decimal)
300	$100 - (6 \times 10^{-80})\%$
366	100%

  - Run experiment: have each person list their birthday and write it on the board
  - Fill in actual table and discuss intuition - distribute the handout here
  - Explain difference between asking for a particular day and any day
  - Explain that it is easier to compute the opposite probability
  - Try to compute probability for only two people, three people, etc. to get a formula
  - Could discuss pigeonhole principle for 366

Questions for class

- How likely do you think it is that at least two people in this room share a birthday?
- How would the birthday problem or formula you found change if the year were longer?

Notes

Application: There's an efficient brute-force method of hacking based on the birthday paradox called the birthday attack.

Assignments

1. Suppose that I have a picture to color in with a dog wearing a hat, a coat, and two shoes (left and right). I have four colors: red, blue, green, yellow; I am allowed to use a color as many times as I want. How many different pictures can I make? If I choose a picture at random, what is the chance that I'll get a picture that's all one color? What if I had five colors? Six? n? What if the dog were wearing more clothes? What do you think makes more of a difference: putting more clothes on the cat, or adding more colors? (Expected length: 1 paragraph)

2. Read this article. What do you think the chance of something like that happening to you is? Try to figure out how you could compute that chance. What do you think the chance of something like that happening to SOMEONE is? Why is this second question different from the first? (Expected length: 1 paragraph.)

3. The "Take 5" New York State Lottery has people choose 5 different numbers 1-40. Groups of five are drawn daily; the order doesn't matter, just the five different numbers picked (so there are no repeats). To win the grand prize you need to match all five on that day. If you pick your numbers randomly, what is the chance that you'll win? (If you get stuck, look up "Binomial coefficient" on Wikipedia.) Do you think anyone has won the grand prize in take 5? Does this align with your feelings about the small number that you computed? (Expected length: 1 paragraph.)

References and resources

Birthday Graph Handout

Article: Couple discovers they were in the same photo 11 years before meeting

Follow-on activities

Reasoning About Probability

Monty Hall Problem